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……In deed and in truth

Mathematical Frameworks for Passive Filter Design for Harmonic Mitigation in Power Supply Networks: A Review

By

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Presentation Outline

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Introduction

- The rapid growth of non-linear loads and inverter-based Renewable Energy Sources (RESs) has worsened Power Quality (PQ) issues in industrial, commercial, utility, and RE networks.
- Harmonic distortion in electricity distribution networks can be repressed utilizing two methods: passive and active filtering [1].
- Passive filtering is the most basic traditional approach for reducing harmonic distortions [1].
- Passive filtering is the most basic traditional, inexpensive and widely utilized approach for reducing harmonic distortions [1].

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Introduction

- Conventional passive filters use a parallel or series combination of passive elements (resistance, R; inductance, L; and capacitance, C) known as RLC components that are designed and tuned to control harmonics [2].

- These three fundamental passive components can be linked in a variety of ways, such that they trap the harmonic content of a certain power network [2-4].

- Some common setups include series, shunt, hybrid, damped, double tuned, high pass, C-type High Pass (HP), and single tuned.

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Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

- The global transition toward RE and electrified infrastructure has significantly increased the deployment of Power Electronic Converters (PECs) in modern power systems [5].

- Solar Photovoltaic (PV) generation, wind energy systems, Electric Vehicle (EV) chargers, and distributed energy resources have been widely integrated into electric power networks [6].

- Although these technologies are essential for sustainable and clean energy development, they introduce considerable harmonic distortion and PQ issues into the grid [7].

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Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

- In renewable-rich power systems, such distortions can reduce system efficiency, accelerate equipment aging, and affect the stable operation of sensitive loads [5-7].

- Errors in control systems and digital meters, flicker, heightened network losses, equipment failure or overheating, and communication system interference are additional induced negative effects of harmonic distortion [5-6].

- To address the harmonic problems in electrical power supply networks, many researchers have proposed diverse solutions - Compensators, regulators, power line conditioners, reactors, use of filters [1-2], [6-7].

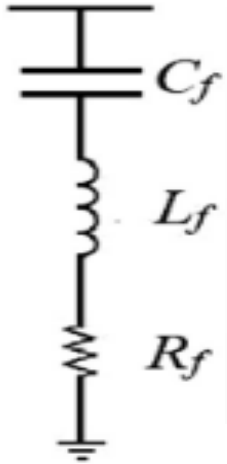
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Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

Single Tuned Passive Filters

- This filter contains a series connection of R, L and C, as shown in Fig. 1. STPFs are broadly utilized to suppress a single harmonic frequency,



STPF equivalent circuit [8-9]

- The design of passive filters depends on three key parameters: quality factor (Q), tuning frequency (ω_t) and reactive power, P_Q [10-11].

- To obtain the values of C, R, and L, Eqs. (1)-(3) are employed

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Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

Single Tuned Passive Filters

$$C = \frac{P_Q}{2\pi f V^2} \quad (1)$$

$$R = \frac{1}{2\pi f Q C} \quad (2)$$

$$L = \frac{RQ}{2\pi f h} \quad (3)$$

- The equivalent impedance of a STPF is given as Eq. (4)

$$\vec{Z}_{stf}(\omega) = R_{stf} + j\omega L_{stf} - j\frac{1}{\omega C_{stf}} \quad (4)$$

- h is the harmonic order, f is the fundamental frequency, ω_1 is the fundamental harmonic angular frequency and the line voltage is V

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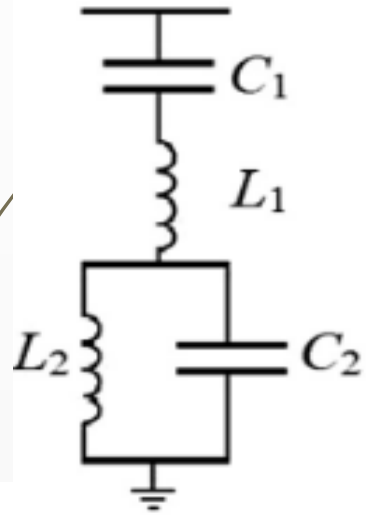


Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

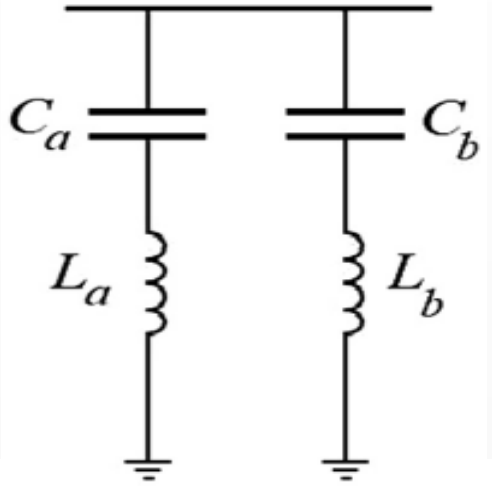
Double Tuned Passive Filters

- DTPFs are used to mitigate two harmonic frequencies

- A standard DTPF comprises of two parts linked in series (the series section and the parallel section).



Standard equivalent circuit of a DTPF [9],[11]



DTPF structure through parallel combination of two STPFs [11]

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Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

Double Tuned Passive Filters

- The capacitance, inductance and impedance equations are expressed in Eqs. (5)-(11)

$$C_1 = C_a + C_b \tag{5}$$

$$L_1 = \frac{L_a L_b}{L_a + L_b} \tag{6}$$

$$L_2 = \frac{(L_a C_a - L_b C_b)^2}{(C_a + C_b)^2 (L_a + L_b)} \tag{7}$$

$$C_2 = \frac{C_a C_b (C_a + C_b)(L_a + L_b)^2}{(L_a C_a - L_b C_b)^2} \tag{8}$$

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Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

Double Tuned Passive Filters

- The capacitance, inductance and impedance equations are expressed in Eqs. (5)-(11)

$$\vec{Z}_{s,dtf}(\omega) = j\omega L_{1,dtf} - j\frac{1}{\omega C_{1dtf}} \tag{9}$$

$$\vec{Z}_{p,dtf}(\omega) = \frac{1}{\frac{1}{j\omega L_{2,dtf}} + j\omega C_{2,dtf}} \tag{10}$$

$$\vec{Z}_{T,dtf}(\omega) = \vec{Z}_{s,dtf}(\omega) + \vec{Z}_{p,dtf}(\omega) \tag{11}$$

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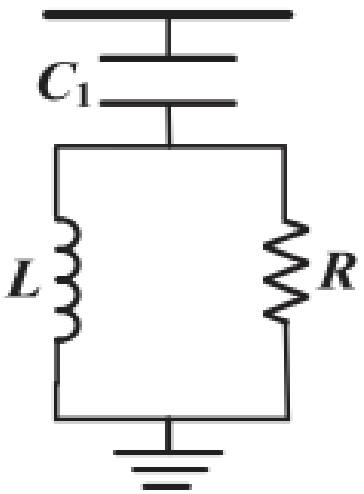


Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

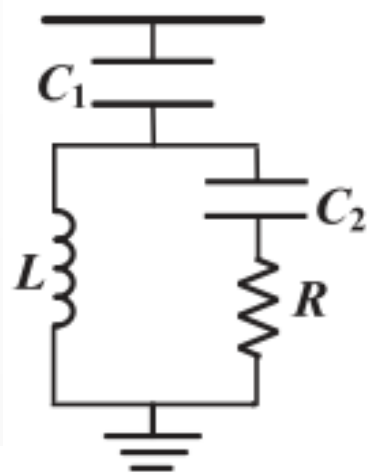
C-Type High Pass Filter

- A C-type HP filter is utilized to attenuate multiple higher order harmonics.

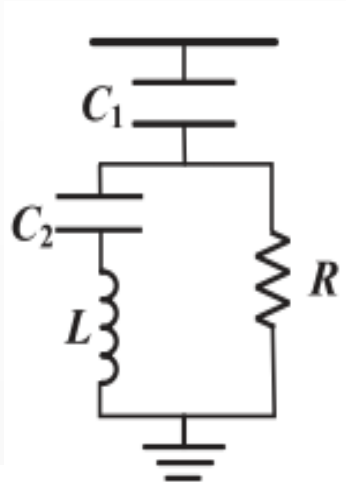
- A HP filter can be constructed using three circuit configurations, namely: 2nd HP filter, 3rd HP filter and C-type filter [12].



2nd order HP filter [12]



3rd order HP filter [12]



C-type HP filter [12]

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Overview of Existing Frequency and Tuning Based-Passive Filter Topologies

C-Type High Pass Filter

- C-type HP filters consist of four elements: C_1, L, C_2, and R. These elements collectively attenuates harmonic distortion in a given network.

$$C_1 = \frac{P_Q}{\omega_1 V^2} \tag{12}$$

$$L = \frac{1}{\omega_1^2 C_2} \tag{13}$$

$$R = \frac{h_t^2 - 1}{\omega_1 h_t \sqrt{(h_t^2 - 1)C_1 C_2 - C_2^2}} \tag{14}$$

$$C_2 \geq \left(\frac{h_t^2 - 1}{h_t^2}\right) C_1 \tag{15}$$

$$Z_{cpf}(\omega) = \frac{1}{j\omega C_1} + \frac{\left(\frac{1}{j\omega C_2} + j\omega L\right) R}{R + j\omega L + \frac{1}{j\omega C_2}} \tag{16}$$



Suggested Topologies for Optimal Harmonic Attenuation

- When these filters (STPF, DTPF and C-type HP filter) are singly integrated in an electric power supply network, they may not adequately damp out the harmonics in such power network.

- For optimal mitigation of various harmonic orders, it is best that a Passive Filter Bank (PFB) is utilized.

- Three PFBs is suggested: Shunt linked dual STPFs and C-type HP filter; Shunt linked dual STPFs and 2nd Order HP filter; and Shunt linked DTPF and C-type HP filter.

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Suggested Topologies for Optimal Harmonic Attenuation

Shunt linked dual STPFs and C-type HP filter

- To design this PFB, the individual admittances of each passive filter are added together to form a unified mathematical framework. This ensures effective absorption of the targeted harmonics in the power network under consideration.

$$Y_{total} = \frac{1}{Z_{stf1}} + \frac{1}{Z_{stf2}} + \frac{1}{Z_{cpf}} \tag{17}$$

$$Y_{total} = \frac{1}{R_{stf1} + j\omega L_{stf1} - j\frac{1}{\omega C_{stf1}}} + \frac{1}{R_{stf2} + j\omega L_{stf2} - j\frac{1}{\omega C_{stf2}}} + \frac{1}{\frac{1}{j\omega C_1} + \frac{\left(\frac{1}{j\omega C_2} + j\omega L\right)R}{R + j\omega L + \frac{1}{j\omega C_2}}} \tag{18}$$

$$Y_{total} = \frac{1}{R_{stf1} + j(X_{L1} - X_{C1})} + \frac{1}{R_{stf2} + j(X_{L2} - X_{C2})} + \frac{1}{-jX_{Cmain} + \frac{R\,(jX_L - jX_{Caux})}{R + (jX_L - jX_{Caux})}} \tag{19}$$

$$Y_{total} = \frac{[Z_{stf2}\ x\ Z_{cpf}] + [Z_{stf1}\ x\ Z_{cpf}] + [Z_{stf1}\ x\ Z_{stf2}]}{Z_{stf1}\ x\ Z_{stf2}\ x\ Z_{cpf}} \tag{20}$$

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Suggested Topologies for Optimal Harmonic Attenuation

Shunt linked dual STPFs and 2nd Order HP filter

- To design this PFB, the individual admittances of each passive filter are added together to form a unified mathematical framework. This ensures effective absorption of the targeted harmonics in the power network under consideration.

$$Y_{total} = \frac{1}{Z_{stf1}} + \frac{1}{Z_{stf2}} + \frac{1}{Z_{2hpf}} \quad (21)$$

$$Y_{total} = \frac{1}{R_{stf1}+j\omega L_{stf1}-j\frac{1}{\omega C_{stf1}}} + \frac{1}{R_{stf2}+j\omega L_{stf2}-j\frac{1}{\omega C_{stf2}}} + \frac{1}{\frac{1}{j\omega C_1}+\frac{j\omega RL}{R+j\omega L}} \quad (22)$$

$$Y_{total} = \frac{1}{R_{stf1}+j(X_{L1}-X_{C1})} + \frac{1}{R_{stf2}+j(X_{L2}-X_{C2})} + \frac{1}{-jX_{C1}+\frac{jRX_L}{R+jX_L}} \quad (23)$$

$$Y_{total} = \frac{[Z_{stf2}\ x\ Z_{2hpf}]+[Z_{stf1}\ x\ Z_{2hpf}]+[Z_{stf1}\ x\ Z_{stf2}]}{Z_{stf1}\ x\ Z_{stf2}\ x\ Z_{2hpf}} \quad (24)$$

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Suggested Topologies for Optimal Harmonic Attenuation

Shunt linked DTPF and C-type HP filter

- To design this PFB, the individual admittances of each passive filter are added together to form a unified mathematical framework. This ensures effective absorption of the targeted harmonics in the power network under consideration.

$$Y_{total} = \frac{1}{Z_{T,dtf}} + \frac{1}{Z_{cpf}} \tag{25}$$

$$Y_{total} = \frac{1}{j\omega L_{1,dtf} - j\frac{1}{\omega C_{1dtf}} + \frac{1}{\frac{1}{j\omega L_{2,dtf}} + j\omega C_{2,dtf}}} + \frac{1}{\frac{1}{j\omega C_1} + \frac{\left(\frac{1}{j\omega C_2} + j\omega L\right)R}{R + j\omega L + \frac{1}{j\omega C_2}}} \tag{26}$$

$$Y_{total} = \frac{1}{jX_{L1,dtf} - jX_{C1,dtf} + \frac{1}{\frac{1}{jXL_{2,dtf}} + jX_{C2,dtf}}} + \frac{1}{-jX_{Cmain} + \frac{R\left(jX_L - jX_{Caux}\right)}{R + \left(jX_L - jX_{Caux}\right)}} \tag{27}$$

$$Y_{total} = \frac{Z_{cpf} + Z_{T,dtf}}{Z_{T,dtf}\ x\ Z_{cpf}} \tag{28}$$



Suggested Topologies for Optimal Harmonic Attenuation

Table 1. Comparison of standalone passive filters and three PFBs – strength and gap

Topology	Strength	Topology wise-gap
STPF	Filters only one specific targeted harmonic frequency	Effective against only a single harmonic frequency
DTPF	Filters only two specific targeted harmonic frequencies	Effective against only two harmonic frequencies
C-type HP filter	Broad spectrum attenuation for high order harmonics	Less effective at filtering specific low-order dominant harmonics
Shunt linked dual STPFs and C-type HP filter (suggested)	✓ Attenuates two distinct harmonic frequencies as well as captures a wide spectrum of high frequency harmonics simultaneously ✓ It is an economical alternative that can be utilized in place of Active Power Filters (APFs)	–
Shunt linked dual STPFs and 2nd Order HP filter (suggested)	✓ Offers a more aggressive slope for high frequency harmonics in addition to mitigating two lower-order harmonic frequencies ✓ It can be utilized in place of APFs	–
Shunt linked DTPF and C-type HP filter (suggested)	✓ More economical to utilize instead of expensive APFs ✓ Mitigates two-specific low-order harmonics simultaneously and then suppresses the remnant high frequency spectrum	–

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Conclusion

- The important role of passive filters in electric power networks cannot be over-emphasized.

- Being a vital player utilized for harmonic mitigation in such networks, the proper design and integration of passive filters will go a long way in enabling the elimination or reduction of harmonic problems in both conventional and RE-based electricity networks.



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